The Possibility of Unicorns and Modal Logic

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Michael Dummett argues, against Saul Kripke, that there could have been unicorns. He then claims that this possibility shows that the logic of metaphysical modality is not S5, and, in particular, that the B axiom is false. Dummett’s argument against B, however, is invalid. I show that although there are number of ways to repair Dummett’s argument against B, each requires a controversial metaphysical or semantic commitment, and that, regardless of this, the case against B is undermotivated. Dummett’s case is still of interest, however, as if his assumptions are correct, S5 has to go, with the natural culprit being S4.

The propositional logic of metaphysical modality has been assumed to be at least as strong as the system T and to be no stronger than the system S5. Given propositional logic, we can axiomatize T as follows:

\[ \text{K: } \Box(P \supset Q) \supset (\Box P \supset \Box Q) \]
\[ \text{T: } \Box P \supset P. \]

And if we add the axioms

\[ \text{S4: } \Box \Box P \supset \Box P \]
\[ \text{B: } \Box P \supset P, \]

we have S5. Equivalently, we can replace S4 and B with
E: ◊P ⊃ □P.¹

In a possible world semantics for modal logic, where ‘◊P’ is true if and only if ‘P’ is true at some accessible world, and ‘□P’ is true if and only if ‘P’ is true at every accessible world, S5 has it that accessibility is an equivalence relation: T ensures reflexivity, S4 transitivity, and B symmetry.

The debate over the correct logic for metaphysical modality has focussed on whether S5 is too strong. Chandler (1976) and Salmon (1989) argue that it is, rejecting S5 because of putative counterexamples to S4. These have been discussed elsewhere.² My aim in this note is to examine the counterexample to S5 proposed by Michael Dummett. Dummett (1993a) argues that there could have been unicorns and that this possibility provides a counterexample to B. Moreover, once B has been rejected, Dummett then goes on to tentatively reject S4 as well:

So far as I know ... no intuitively plausible example of a failure of the accessibility relation to be both transitive and symmetrical has ever been proposed. I aim in this essay to suggest one (Dummett 1993a: 331).

Dummett’s argument, although well known, is little discussed, especially given the import of its conclusions. What discussion there has been (Reimer (1997), and Rumfitt (2010)) has focussed on whether Dummett is correct that there could have been unicorns. What I show below is that even if we grant that there could have been unicorns, Dummett’s argument fails

¹ Both systems also employ the rule of Necessitation: if P is a theorem, so is □P. Necessitation combined with classical logic results in the necessary existence of everything (∀y ∃x x=y). One can embrace this result with Williamson (1998), or else avoid it by adopting a free logic (Rumfitt (2003)), or by restricting Necessitation (Wiggins (2003)).
and so we should not agree with Rumfitt (2001: 322 n23) that B “has been seriously challenged” by Dummett.

In §1 I outline Dummett’s argument against B and S4. I then show (§2) that Dummett’s argument is invalid, but that his case is still of interest as it does provide a counterexample to S5, and albeit one which is naturally thought of as a counterexample to S4. Finally, in §3, I argue that we can restore Dummett’s case against B, but that such a case is undermotivated and relies on further controversial metaphysical or semantic claims. The Dummettian case against B is un compelling, even granting the possibility of unicorns.

1. Dummett’s Example

There is a possible world, u, where there are creatures resembling the unicorns of the myth, which belong, like deer, to the order Artiodactyla. Similarly, there is a world, v, where there are creatures resembling the unicorns of the myth, which belong, like horses, to the order Perissodactyla. But are the creatures of u and v are unicorns? Kripke (1980) suggests that semantic considerations mean that there could not have been unicorns, and so the creatures of u and v are not unicorns.³ Dummett (1993a) argues against Kripke’s semantic thesis and claims that the creatures of both u and v are unicorns. Let’s grant Dummett this claim for the sake of argument.

In order to generate his counterexample to B, Dummett also relies on the following claim:

³ I say ‘suggest’ as it is not entirely clear what Kripke’s (1980) view is. Regarding the possibility of unicorns, he says that “this is an example of something I think is not the case” (p. 24). But he immediately caveats this claim: “Perhaps according to me the truth should not be put in terms of saying that it is necessary that there should be no unicorns, but just that we can’t say under what circumstances there would have been unicorns” (p. 24) When he returns to the issue, it seems as if the same mixed message is present as he says the claim is “a metaphysical thesis that no counterfactual situation is properly describable as one in which there would have been unicorns” (p.156 my underlining).
In world u, any animal, to be a unicorn, must have the same anatomical structure as the unicorns in u, and hence, in particular, must belong to the order Artiodactyla. ... u is a world in which it holds good that unicorns are necessarily of the order Artiodactyla (Dummett 1993a: 346).

Given these two assumptions – the possibility of unicorns of the order Artiodactyla and of the order Perissodactyla, and the necessity of order membership – Dummett argues that B is to be rejected. In particular, Dummett thinks that the actual world, w, is not accessible from either u or v:

Since a proposition necessarily true in u is possibly false in w, w cannot be possible relatively to u, although u is possible relatively to w. The relation of relative possibility (accessibility) is therefore not symmetrical (Dummett 1993a: 346).

As noted above, in a possible world semantics for modality, the symmetry of the accessibility relation is equivalent to B. So what Dummett is arguing for in the quote above, is the falsity of the B axiom.

Although Dummett does not think that w is accessible from u or v, he does want to allow that unicorns do not exist of necessity at u or v. So although w is not accessible from u or v, there are worlds accessible from u and v, s and t respectively, where there are no unicorns. Moreover, Dummett thinks that such worlds may differ from w only in modal terms: “in s and t all non-modal propositions hold good that hold in the actual world” (Dummett 1993a: 347).

Now not only does Dummett reject B, he also tentatively rejects S4. This rejection relies on the claim that the modal supervenes on the non-modal, which we can formulate as
Modal Supervenience: For any accessible $w^*$, if $w^*$ and $w$ are the same with respect to non-modal truths, then they are the same with respect to modal truths.

Since $s$ differs from $w$ only in modal truths – unicorns could have been of the order Perissodactyla at $w$, but not at $s$ – then given Modal Supervenience, $s$ and $w$ cannot be accessible from one another other. As a result, since $u$ is accessible from $w$, and $s$ is accessible from $u$, it follows that accessibility is not transitive. That is, the possible possibility of $s$ is not possible at $w$. As a result, $S4$ is false.

Dummett’s rejection of $S4$ is tentative as he does not commit to Modal Supervenience. Minimally what Dummett could have claimed is that not only if $B$ false, but also that the conjunction of Modal Supervenience and $S4$ is to be rejected, and additionally that the following stronger supervenience thesis is false:

Modal Supervenience*: For any $w^*$, accessible or not, if $w^*$ and $w$ are the same with respect to non-modal truths, then they are the same with respect to modal truths.

Dummett’s example, then, purports to have serious consequences. In the next section I argue that this is indeed the case, although they are not the consequences Dummett claims.

2. What Dummett’s Example Really Shows

Dummett’s argument against $B$ is invalid. Dummett reasons that “Since a proposition necessarily true in $u$ is possibly false in $w$, $w$ cannot be possible relatively to $u$, although $u$ is possible relatively to $w$ (Dummett 1993a: 346). But this is a non-sequitur. What actually follows is that $S5$ is to be rejected. Let us call the proposition that Dummett is concerned with ‘$P$’. Dummett claims that $P$ is necessarily true in $u$, so that at $w$ we have $\Box \Diamond P$. He also claims
that this proposition is possibly false in w, namely \( \Diamond \sim P \). But what follows from the conjunction \( \Diamond \Diamond P \land \Diamond \sim P \) is the falsity of

\[ E: \Diamond \Diamond P \supset \Box \sim P. \]

But this leaves it open whether it is B or S4 which is to be rejected. For \( \Diamond \Diamond P \) to falsify B we need not just that \( \Diamond \sim P \), but the stronger claim \( \sim P \).

To put things in possible world terms, given Dummett’s premises, \( v \) must be inaccessible from \( u \), since the unicorns of \( u \) are necessarily of the order Artiodactyla, whereas in \( v \) there are unicorns not of the order Artiodactyla; rather, at \( v \) the unicorns are of the order Perissodactyla. So, if \( v \) were accessible from \( u \), then at \( u \) we would have something of the form \( \Box \sim P \land \Diamond \sim P \), which is a contradiction given the interdefinability of ‘\( \Box \)’ and ‘\( \Diamond \)’. Symmetrical considerations bar the accessibility of \( u \) from \( v \). Now given that both \( u \) and \( v \) are accessible from the actual world, \( w \), the mutual inaccessibility of \( u \) and \( v \) shows only that the accessibility relation cannot be both symmetric and transitive, which is to say S5 must go. If, as Dummett claims, the accessibility relation is not symmetric, so that \( w \) is not accessible from either \( u \) or \( v \), then this is enough to prevent \( u \) and \( v \) being accessible from one another via \( w \). In this case, the transitivity of the accessibility would not be threatened. On the other hand, if the accessibility relation is not transitive, then nothing prevents \( w \) being accessible from \( u \) and from \( v \), since this only entails that \( u \) and \( v \) are accessible from one another assuming the accessibility relation is transitive. So if we accept Dummett’s case, then for all that has been said so far, it could be that transitivity and S4 are to be given up, rather than symmetry and B.

Moreover, not only is Dummett’s argument invalid, we can see that it is natural to think that his case furnishes no reason to reject B. Letting U, A, and P stand for ‘is a unicorn’, ‘is a
member of the order Artiodactyla’, and ‘is a member of the order Perissodactyla’, Anderson
(2012: 303) construes Dummett’s claim about the possible worlds u and v as

1. ◊∃x (Ux ∧ □ ∀x (Ux ⊃ Ax))

and

2. ◊∃x (Ux ∧ □ ∀x (Ux ⊃ Px))

respectively.⁴

By S5 it follows from (1) that

3. □ ∀x (Ux ⊃ Ax)

which, on the assumption that the orders Artiodactyla and Perissodactyla are mutually
exclusive, is inconsistent with

4. ◊∃x (Ux ∧ Px),

which follows from (2). So given Dummett’s assumptions, S5 has been shown to be false.
But by B, all (1) yields is

5. ∀x (Ux ⊃ Ax)

⁴ This seems like a fair construal of Dummett as Dummett makes claims about “any unicorn” in the
quote about order membership above. We consider alternative renderings below.
which is vacuously true at w, and is not inconsistent with (4) or with (2). So Dummett’s example only shows that S5 is false, not that B is.

Indeed, since the minimum needed to secure consistency with Dummett’s assumptions is that u and v are mutually inaccessible, it is gratuitous, without further argument, to make w inaccessible from either u or v. So it is natural to take Dummett’s case as showing that it is transitivity and S4 that have to go, not symmetry and B. Further, if we accept this, there is no reason to suppose that there are worlds such as s and t, which differ from w only in modal truths. Dummett wants to allow that unicorns exist contingently at u and v. He thinks he needs to posit s and t to secure this result because w is inaccessible from u and v. But as we have just seen, there is, as yet, no reason to think that w is inaccessible from u and v. Given this, w can account for the contingent existence of unicorns at u and v. Therefore, we have no need for s and t after all. As a result neither Modal Supervenience* nor Modal Supervenience are threatened by Dummett’s example.

3. Can Dummett’s Case Against B be Fixed?

The above discussion assumed, following Anderson, that the necessity of the order membership of unicorns at u is to be formulated as $\Box \forall x (Ux \supset Ax)$. But perhaps we can capture this thought, or a corollary of it, with the necessary truth of one of the following (where ‘u’ is a singular term putatively referring to the kind unicorn):$^5$

6. $\forall x (Ux \supset \neg Px)$

7. $\neg(\lambda x.Px)(u)$

8. Au

$^5$ Context will determine whether ‘u’ is a singular term putatively referring to a kind, or whether it denotes a possible world. There is also a harmless ambiguity in allowing ‘A’ and ‘P’ stand for a predicate which applies to individual animals, and also as a predicate which applies to kinds.
By appealing to the necessity of one of (6)-(9), instead of $\forall x (Ux \supset Ax)$, perhaps we can show that w is inaccessible from u after all. So the question is, are any of (6)-(9) necessary at u, but not true at w? Before we examine this question we should note that if unicorns are to be contingent existents at u, then if any of (6)-(9) are necessary at u, they will also be true at worlds such as s which are accessible from u and where unicorns fail to exist. This means that if w is inaccessible from u in virtue of the falsity of one of (6)-(9) at w, then w and s differ in non-modal terms, since one of (6) to (9) is true at the latter, but not at the former. So the possibility of unicorns does not require us to consider pairs of worlds which differ only modally. Consequently, this revised case poses no threat to Modal Supervenience* or to Modal Supervenience.

Let's return to (6)-(9) to see if they can be used to reinstate the case against B. Given that there are no unicorns at w, (6) is vacuously true at w. As a result, the necessity of (6) at u provides no reason to deny that w is accessible from u, and so poses no threat to B. (7)-(9) are potentially more problematic for B, although the issues are more complex. In order for any of (7)-(9) to be necessarily true at u, they must be true at worlds accessible from u where there are no unicorns, such as s. Moreover, if w is to be inaccessible from u, they must also fail to be true at w. There are several ways in which (7)-(9) can fit this profile, but all of them require controversial semantic or metaphysical assumptions. To see this we should ask what, if anything, 'u' refers to at worlds where there are no unicorns, such as w and s. There seem to be the following salient possibilities:

A. ‘u’ does not refer

B. There is something to which ‘u’ refers, namely a non-existent kind
C. Although there is nothing to which ‘u’ refers, there could have been something to which ‘u’ actually refers: ∃x(∀‘u’ refers to x)⁶

D. Even though there are no unicorns, ‘u’ refers to the existent kind unicorn

Obviously, if we endorse (D), then we have to say that natural kinds can exist at worlds even when there are no instances of that kind at that world. I offer no argument that this is not possible, but it is a controversial metaphysical claim that an opponent of B may not wish to commit to.⁸

If instead we endorse one of (A)-(C), and say that (8) and (9) are necessarily true at u, and so true at s, then we have a failure of

Serious Actualism: Necessarily, a subject-predicate sentence, ‘Fa’, is true at w only if a exists at w.

Personally, I do not see how Serious Actualism could fail: how can an object be any way, if there is no such object. Of course, the Meinongian advocate of (B) may posit distinction between existence and being, but as this distinction is obscure, it does not seem that proposal (B) is much better off in this regard than (A) and (C). Moreover, we should note that the necessity of (8) or of (9) at u requires a particular kind of failure of Serious Actualism:

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⁶ Both (B) and (C) have ‘u’ designating a non-existent. The difference between them is that whereas (B) has quantifiers ranging over non-existents, (C) does not. As we are supposing the non-existents in questions could have existed, then (B) employs possibilist quantifiers whereas (C) employs actualist quantifiers.

⁷ In a possible world semantics for modality we might gloss (C) as saying that ‘u’ refers to a mere possible. But it is not clear that we can take the possible world semantics seriously if we employ a variable domain semantics. As Bacon (2013: 9) puts it “Since Pegasus doesn’t exist, the sentence ‘Pegasus does not exist’ must be true according to the intended model. But if the intended model is of the Meinongian sort described, then there is some object, the object the model assigns to ‘Pegasus’, which does not belong to the domain of the model. Thus the model in question can’t be the intended model since it fails to accurately capture the actual range of our quantifiers”. See also Williamson (1998: 263).

⁸ I argue elsewhere (Walters 2013) that some types, such as repeatable artworks, can exist even when they have no tokens. The considerations in favour of that claim, however, do not entail that species can exist without members.
some may be happy to grant exceptions to Serious Actualism in the case of modal or identity properties, but these exceptions are of no use in this case. My purpose, however, is not to argue for Serious Actualism here, but merely to note that any case against B which rests on one of (A)-(C) and on the necessity of (8) or of (9) at u, is committed to the denial of Serious Actualism.\footnote{Assuming, with Dummett, that unicorns do not necessarily exist at u, and so there are worlds like s which are accessible from u.}

(7), however, is not a subject-predicate sentence, but rather the negation of one. So an appeal to the necessity of (7) at u does not entail the denial of Serious Actualism. Moreover, if (i) (λx.Px)(u) is false at u and s, (ii) (λx.Px)(u) is neither true nor false at w, and (iii) the negation of (λx.Px)(u) fails to be true at w, then (7) can be necessarily true at u and yet not true at w, without either a violation of Serious Actualism or the possibility of natural kinds without instances. But each of (i)-(iii) are themselves substantive semantic commitments. Moreover, in order for (7) to have the correct profile for a case against B, (i) and (ii) both have to be true. That is, (λx.Px)(u) must be false at s and yet neither true nor false at w. But what could motivate this picture? One might reasonably think that the absence of an existent referent automatically robbed a subject-predicate sentence of the ability to express a truth or falsehood, or instead that any thought expressed by such a sentence was automatically false. But what grounds could there be for allowing that some such sentences were false and some neither true nor false?

Here is one way to motivate such a ground. Let us follow Fitting and Mendelsohn (1998: 190) in “distinguishing terms that designate non-existent objects from terms that do not designate”. Again we might follow Fitting and Mendelsohn (1998: 231) in claiming “that there are two reasonable approaches” to take to subject-predicate sentences containing terms that do not designate. Namely, to treat them as expressing no thought, and consequently as neither true nor false, or else as treating them all as automatically false. Let’s now depart
company from Fitting and Mendelsohn and take the former route: every thought requires an object, and so sentences without the requisite objects are neither true nor false. Again let’s depart from Fitting and Mendelsohn, and embrace Serious Actualism so that subject-predicate sentences whose singular term refers to a non-existent are false. So here we have a motivation for a free logic which does not fall into the traditional trichotomy of positive, negative, and neutral free logics, since subject-predicate sentences containing singular terms referring to non-existents are automatically false, whereas subject-predicate sentences containing singular terms which are empty are neither true nor false.\textsuperscript{10}

But in order for this to provide a case against B, however, it must be that whereas ‘u’ refers at s, so that $(\lambda x. P x)(u)$ is false, it fails to refer at w, so that $(\lambda x. P x)(u)$ is neither true nor false. But what could justify this claim? Again taking our lead from Fitting and Mendelsohn, we might think that within the framework above it is only the names of impossible objects that genuinely fail to refer. In the case of possible objects, names refer to possibilia. But at both s and w, the kind unicorn is a possibile, and so, on this view, ‘u’ refers at both. So although this unorthodox form of free logic can be motivated, it is of no use in mounting a case against B.

Any of the above cases against B, then, rely on further controversial assumptions. Furthermore, I think that we can see that any such rejection of B is unattractive. We said above that s and w have to differ non-modally if we are to have a counterexample to B, since one of (7)-(9) must be true at s but not true at w. But it does not follow from this that s and w differ in what is true at them with respect to existents. Moreover, it seems that there could be worlds like s and w which do not differ with respect to the existent: whatever we have to add to w to get u, we can seemingly subtract from u to get s. If that is correct, then although s and w are intrinsic duplicates of one another as far as existents goes, they differ with respect

\textsuperscript{10} The former class of subject-predicate sentences alone requires a free logic, since, assuming Serious Actualism, we cannot go from $\forall x (x=x)$ to $a=a$ when ‘a’ names a non-existent.
to non-modal truths concerning unicorns, which do not exist at either $s$ or $w$. But how could this be? Of course, to register my incredulity is not to offer an argument, but nevertheless this failure of non-modal truth to supervene on truths concerning the existent seems unattractive, and at the very least is a substantial commitment of any Dummettian case against B. Similarly, even if the kind unicorn exists at $s$ and $w$, as option (D) has it, it is hard to see, in the absence of some further intrinsic difference between $s$ and $w$, how any of (7)-(9) could differ in truth value between $s$ and $w$.\footnote{I think that Dummett himself would not have considered options (B) and (C) since he claims that “we need to take the semantic role of an empty name to consist in its having no bearer, not in its having a non-existent or otherwise peculiar one” (1983: 295). Moreover, it seems he would not be attracted to reviving his argument assuming (A). This is because he appears to subscribe to something like a negative semantics for free logic, and so (7)-(9) would each have to have the same truth value at $s$ as they did at $w$. However, because he thinks that the distinction between internal and external negation is “misconceived” (1983: 296) Dummett doesn’t characterise a negative free logic in the usual manner, like, for example, Evans (1982: 37). Nonetheless, Dummett wants “Guinevere was not married to King Arthur” to come out false if Guinevere did not exist but true if she existed but Arthur did not; we have therefore to read it as meaning “For some $X$, $X$ was Guinevere and it is not the case that, for some $Y$, $Y$ was Arthur and $X$ was married to $Y$” (1983: 294). The generalization of this thought gives something like a negative semantics for free logic.}

Given the oddity of any of the above pictures, perhaps we should consider whether we need to claim that any of (7)-(9) is both necessarily true at $u$ and yet fails to be true at $w$? Starting with (7), it seems reasonable to conclude that (7) is necessary at $u$, assuming that it can be true at worlds where there are no unicorns, given that unicorns are necessarily of the order Artiodactyla at $u$. But precisely because it can be true at such worlds, there doesn’t seem to be any reason to deny that it is true at $w$.

What about (8) and (9)? Here, I suggest, it is not clear that we want to grant that they are necessarily true at $u$. If we take the necessity of identity to be given by

\begin{equation}
10. \ a=b \supset \Box a=b,
\end{equation}

then we might think by analogy that the necessity of order membership is to be given by
11. Au ⊃ □Au

or

12. (λx.¬Px)(u) ⊃ □(λx.¬Px)(u)

depending on whether we are dealing with (8) or (9) respectively.

And given that at u, the antecedents of (11) and (12) are true, (11) and (12) yield the necessity of (8) and (9) at u as required.

But when dealing with contingent existents we may prefer to formulate our claims of necessity with existential riders. So instead of formulating the necessity of identity as (10), we would formulate it as

13. a=b ⊃ □(∃x (x=a ∨ x=b) ⊃ a=b).

Similarly, instead of (11) and (12) the necessity of order membership can be captured by

14. Au ⊃ □(∃x x=u ⊃ Au)

and

15. (λx.¬Px)(u) ⊃ □(∃x x=u ⊃ (λx.¬Px)(u))

But all we get from the truth of (14) and (15) and their antecedents at u and is
16. $\Box(\exists x. x=u \supset Au)$

and

17. $\Box(\exists x. x=u \supset (\lambda x. \neg Px)(u))$

at $u$, rather than the necessity of (8) and (9). But the accessibility of $w$ from $u$ is consistent with the truth of (16) and (17) at $u$, on the assumption that the kind unicorn does not exist in the world where there are no unicorn. In order to get the necessity of (8) or (9), we need $\Box \exists x. x=u$, which is not at all obvious. What this shows, then, is that we can commit to the necessity of order membership at $u$ without committing to the necessary truth of (8) or (9). As a result, there is no compelling case against B from the necessity of order membership and the possibility of unicorns.

4. Conclusion

Dummett’s argument against B and the combination of Modal Supervenience and S4 relies on the controversial claim that there could have been unicorns. But even if we grant this claim, Dummett’s argument is invalid. His example is not without interest, however, as it shows that S5 is invalid, and suggests that it is S4 that should go. Although, there are ways of restoring Dummett’s case against B, these all rely on controversial metaphysical or semantic claims about natural kinds. Furthermore, since these cases against B rely on particular ways of capturing the necessity of order membership which are not mandatory, these cases against B are undermotivated. There is, then, no compelling reason to accept that there is a proposition necessarily true at $u$, which fails to be true at $w$, and so we have
no clear Dummettian counterexample to B. The upshot is that whereas the advocate of S5 must dispute Dummett’s premises, the proponent of B need not.\textsuperscript{12}

References


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